

Exercício 1

a)

$$e_1 m \log m \leq \log n! \leq e_2 m \log m$$

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~~$$e_2 m \log m \geq \log n!$$~~

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~~$$e_1 m \log m \leq \log n!$$~~

$$\exists e_1, e_2, m_0 : e_1 \cdot m \log m \leq \log n! \leq e_2 \cdot m \log m$$

$$e_1 \cdot \log n^m \leq \log n!$$

$$e_2 \cdot \log n^m \geq \log n!$$

$$\log n! \geq e_1 \cdot \log n^m$$

$$\forall m \geq 1 \quad e_1 = 1$$

$$e_2 \cdot \log n^m \geq \log n!$$

$$\forall m \geq 1 \quad e_2 = 1$$

b)

$$\log(n!/2^m) = \Theta(m \log m)$$

$$\exists e_1, e_2, m_0 : e_1 \cdot m \log m \leq \log(n!/2^m) \leq e_2 \cdot m \log m$$

$$\log \frac{n!}{2^m} = \log(n! \cdot 2^{-m}) \quad \log n! + \log 2^{-m}$$

$$\log n! - m \log 2$$

$$\log n! - m \geq e_1 \cdot m \log m$$

c) $m \log \sqrt{m} = \Theta(m \log m)$

$$\exists e_1, e_2, m_0 : e_1 \cdot m \log m \leq m \log \sqrt{m} \leq e_2 \cdot m \log m$$

$$m \log \sqrt{m} \geq e_1 \cdot m \log m$$

$$m = 1 \quad e_1 = \frac{1}{2}$$

$$\frac{1}{2} m \log m \geq e_1 \cdot m \log m$$

Quelquante à prouver sur e_2